# Faculty of Engineering and Technology Department of Electrical and Computer Engineering Network Analysis I, ENEE2304 <br> FinalExam 

Date: Tue Jun, 122018
Time: 150 minutes
Name:
Student \#:

## Opening Remarks:

- This is a 150 -minute exam. Calculators are allowed. Mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and equations in details to get full credit.
- The exam 6 questions 2 pages.


## Problem 1 (16 points)

The $Z_{L}$ in the circuit was chosen to achieve maximum power transfer:
a) Find the value of $Z_{L}$
b) Find the maximum power that can be delivered to $Z_{L}$


$$
\begin{gathered}
Z_{t h}=\frac{V_{O C}}{I_{S C}} \\
\frac{V_{O C}-100 \angle 60}{50}+\frac{V_{O C}}{-25 j}-0.02\left(100 \angle 60-V_{O C}\right)=0 \\
V_{O C}=\frac{4 \angle 60}{0.04+0.04 j}=\frac{4 \angle 60}{0.0566 \angle 45}=70.7 \angle 15 \mathrm{~V} \\
I_{S C}=\frac{100 \angle 60}{50}+0.02 * 100 \angle 60=4 \angle 60 \\
Z_{t h}=\frac{V_{O C}}{I_{S C}}=\frac{70.7 \angle 15}{4 \angle 60}=17.7 \angle-45 \\
Z_{L}=Z_{t h}^{*}=17.7 \angle 45 \\
P_{L m a x}=\frac{V_{t h}^{2}}{8 R_{t h}}=\frac{(70.7)^{2}}{8 * 17.7}=35.3 W
\end{gathered}
$$

## Problem 2 ( 17 points)

The switch in the circuit has been open for a long time. At $t=0$ it becomes closed. Find
a) Find $\mathrm{V}_{\mathrm{c}}(0+)$ and $\mathrm{i}_{\mathrm{c}}(0+)$ ?
b) Show that type of damping is over damped?
c) For $t \geq 0$ find $\mathrm{i}_{1}(\mathrm{t})$ ? -note find $\mathrm{V}_{\mathrm{c}}(\mathrm{t})$ is useful-


$$
\begin{gathered}
V_{C}\left(0^{+}\right)=V_{C}\left(0^{-}\right)=0 V \\
i_{C}\left(0^{+}\right)=i_{L}\left(0^{-}\right)-\frac{V_{C}\left(0^{+}\right)}{R}=i_{L}\left(0^{-}\right)=4 A
\end{gathered}
$$

b) Show that type of damping is over damped?

$$
\begin{gathered}
\alpha=\frac{1}{2 R C}=\frac{1}{2(40)\left(25 * 10^{-8}\right)}=50 \mathrm{krad} / \mathrm{s} \\
\omega=\frac{1}{\sqrt{L C}}=\frac{1}{\sqrt{0.02 * 5 * 10^{-5}}}=40 \mathrm{krad} / \mathrm{s} \\
s_{1,2}=-\alpha \pm \sqrt{\alpha_{o}^{2}-\omega_{o}^{2}}=-50 k \pm \sqrt{50 k^{2}-40 \mathrm{k}^{2}}=-70 k,-30 k
\end{gathered}
$$

c) For $t \geq 0$ find $\mathrm{i}_{\mathrm{L}}(\mathrm{t})$ ? ( 15 marks)

$$
\begin{gathered}
V_{c}(t)=A_{1} e^{s_{1} t}+A_{2} e^{s_{2} t} \\
V_{c}\left(0^{+}\right)=A_{1}+A_{2}=0 \\
A_{1}=-A_{2} \\
\frac{d V_{c}}{d t}\left(0^{+}\right)=s_{1} A_{1}+s_{2} A_{2}=\frac{i_{C}\left(0^{+}\right)}{C}=\frac{4}{25 * 10^{-8}} \\
\left(s_{1}-s_{2}\right) A_{1}=(-30 k--70 k) A_{1}=\frac{4}{25 * 10^{-8}} \\
A_{1}=400 \\
A_{2}=-400 \\
V_{c}(t)=400 e^{-30000 t}-400 e^{-70000 t} \\
i_{1}(t)=\frac{V_{c}(t)}{80}=5 e^{-3000}-5 e^{-7000}
\end{gathered}
$$

## Problem 3 (17 points)

For the circuit shown below find the h-parameters if the $\omega=1 \mathrm{krad} / \mathrm{s}$ showing analysis.


$$
\begin{aligned}
& V_{1}=h_{11} I_{1}+h_{12} V_{2} \\
& I_{2}=h_{21} I_{1}+h_{22} V_{2}
\end{aligned}
$$

For $V_{2}=0$


For $i_{1}=0$


$$
h_{12}=0
$$

$$
\begin{gathered}
\frac{V_{2}}{I_{2}}=\frac{-20 j}{20-j} \\
h_{22}=\frac{I_{2}}{V_{2}}=\frac{20-j}{-20 j} \\
V_{1}=60 I_{1} \\
I_{2}=-2.5 I_{1}+\frac{20-j}{-20 j} V_{2}
\end{gathered}
$$

## Problem 4 ( 18 points)

For the below filter find the center frequency, the cutoff frequencies, bandwidth $\beta$, and Quality factor $Q$.

center frequency $\omega_{o}=2 \pi f_{o}$

$$
\begin{gathered}
\omega_{o}=\frac{1}{\sqrt{L C}}=\frac{1}{\sqrt{2.5 \times 10^{-9} \times 1 \times 10^{-6}}}=20 \mathrm{Mrad} / \mathrm{sec} \\
f_{o}=3.18 \mathrm{MHz} \\
Q=\frac{\omega_{o}}{\beta}=\frac{20 \mathrm{M}}{12.6 \mathrm{M}}=1.59
\end{gathered}
$$

cutoff frequencies

$$
\begin{gathered}
\omega_{c 1}=-\frac{R}{2 L}+\sqrt{\left(\frac{R}{2 L}\right)^{2}+\frac{1}{L C}}=-\frac{\beta}{2}+\sqrt{\left(\frac{\beta}{2}\right)^{2}+\omega_{o}^{2}} \\
=-12.6 M+\sqrt{(12.6 M)^{2}+(20 M)^{2}}=14.7 \mathrm{Mrad} / \mathrm{sec} \\
f_{c 1}=2.33 M H z \\
\omega_{c 2}=\frac{\beta}{2}+\sqrt{\left(\frac{\beta}{2}\right)^{2}+\omega_{o}^{2}}
\end{gathered}
$$

$$
\begin{gathered}
=12.6 M+\sqrt{(12.6 M)^{2}+(20 M)^{2}}=27.3 \mathrm{Mrad} / \mathrm{sec} \\
f_{c 1}=4.34 M H z
\end{gathered}
$$

## Problem 5 (16 points)

Find the capacitor value that improve the system power factor to $\mathbf{9 5 \%}$ for $\mathbf{5 0 H z}$ source


$$
\begin{gathered}
Q_{\text {ini }}=100 k * \tan \left(\cos ^{-1} 0.7\right)=102 k \\
Q_{f i n}=100 k * \tan \left(\cos ^{-1} 0.95\right)=32.87 k \\
Q_{c}=Q_{f i n}-Q_{i n i}=102 k-32.87 k=69.15 k \\
C=\frac{Q_{c}}{V_{r m s}^{2}}=\frac{69.15 k}{2 \times 50 \pi \times 480^{2}}=955 \mu F
\end{gathered}
$$

## Problem 6 ( 16 points)

Find the current $i_{x}(t)$


DC off


$$
\begin{gathered}
-12 \angle 0+10 I_{x 1}+140 I_{x 1}=0 \\
I_{x 1}=80 \angle 0 m A=80 \cos 6 t \quad m A
\end{gathered}
$$

AC off


$$
\begin{gathered}
-20+160 I_{x 2}=0 \ldots .1 \\
I_{x 2}=\frac{1}{8} A \\
I_{x}=I_{x 1}+I_{x 2} \\
I_{x}=(80 \cos 6 t+125) m A
\end{gathered}
$$

## Good Luck

